Uniqueness Aspects of Model-Updating Procedures

T. Janter* and P. Sas†
Katholieke Universiteit Leuven, Leuven, Belgium

Application of model-updating techniques to obtain improved finite-element models for dynamic behavior computations is often associated with the critical observation that model-update techniques can yield completely different models. In the past, this has been a reason for avoiding the use of such procedures. In order to obtain more insight into this phenomenon, a theoretical study about the uniqueness aspects of model update results is presented. A first problem is treated in which the equivalence of stiffness and mass modifications is evaluated. In a second step, the existence of equivalent finite-element models is investigated. As a result, practical conclusions about the quality of model-update results will be drawn.

Nomenclature

$[H(\omega)]$ = frequency response function matrix
j = $\sqrt{-1}$
$[K]$ = stiffness matrix
$[\Delta K]$ = modification of stiffness matrix
$[M]$ = mass matrix
$[\Delta M]$ = modification of mass matrix
m_k = kth modal mass
λ_k = kth complex eigenfrequency
ν_k = kth damped eigenfrequency
$\{\psi_k\}$ = kth mode vector
ω_k = kth undamped eigenfrequency
$*$ = complex conjugate

I. Introduction

APPLICATION of finite-element models for computation of dynamic behavior is becoming more and more common in industry. However, practical use of these techniques has made it clear that a thorough verification of the accuracy of these models is still necessary. A realistic method is to compare finite-element results (eigenfrequencies, mode shapes, generalized masses, etc.) with information obtained from high-quality modal survey tests. If unacceptable discrepancies occur, a model-updating step can be considered in which the model parameters are adjusted for optimal improvement of correlation. These updated models can then serve as a basis for further analysis of the design under study (coupling, modification prediction, etc.)

Several approaches have been suggested to perform the model-updating task. A classification of these models based on the algorithm applied is suggested by Heylen.[1] Another classification is based on the type of updating parameters selected: individual mass and/or stiffness matrix elements,[2-6] submatrices of the global mass and/or stiffness matrices,[7,8] or the finite-element input data.[9]

One of the essential obstacles to the increased use of these techniques is the observation that, in general, it is very difficult to interpret the update of results or that the updated parameters may lose any physical meaning. Difficulties of this type are related to uniqueness aspects of the model-updating approach. Therefore, this study aims at obtaining better insight into this uniqueness quality.

Starting from the definition of "identical dynamic behavior," two related problems are treated. First, a study of equivalent mass and stiffness modifications is performed. Next, the related problem of "equivalence of finite-element models" is considered. From this study, interesting conclusions for practical application of model-updating techniques can be drawn related to the necessity of updating mass and/or stiffness parameters simultaneously, and about the importance of the dimensionality of the vector space of updating parameters as a controlling variable of the updating process.

II. Identical Dynamic Behavior

Mathematically, the linear dynamic behavior of mechanical structures can be evaluated by means of the frequency-response function matrix $[H(\omega)]$.

$$[H(\omega)] = \sum_{k=1}^{n}\left(\frac{a_k\{\psi_k\}\{\psi_k\}^t}{j\omega-\lambda_k}+\frac{a_k^*\{\psi_k^*\}\{\psi_k^*\}^t}{j\omega-\lambda_k^*}\right) \tag{1}$$

where

$$a_k = \frac{1}{2j\nu_k m_k} \tag{2}$$

From these formulas, we can conclude that one way to specify identical linear dynamic behavior of two structures is to require three qualities:

R1) The complex eigenfrequencies are the same.

R2) The mode shapes are the same.

R3) The modal masses are the same, for identical scaling of the eigenvectors. In this study, the "identical dynamic behavior" requirements will be referred to as R1, R2, and R3.

III. Equivalence of Stiffness and Mass Modifications

In order to develop equations for identification of equivalent stiffness and mass modifications, only requirements R1 and R2 (Sec. II) are considered initially. Later on, the implications of introducing requirement R3 are discussed.

Assume an initial, undamped, finite-element model to be characterized by its structural matrices $[K]$ and $[M]$, and having dynamic characteristics $\omega_{k,o}$ and $\{\psi_{k,o}\}$, $k=1,\ldots,n$. (Here n is the total number of degrees of freedom of the fi-

Received Jan. 16, 1989; revision received July 4, 1989.

*NFWO Senior Research Assistant, Department of Mechanical Engineering.

†NFWO Research Associate, Department of Mechanical Engineering.

nite-element model.) The original eigenvalue problem is given as

$$\left(-\omega_{k,o}^2[M]+[K]\right)\{\psi_{k,o}\}=\{0\}, \qquad k=1,\ldots,n \tag{3}$$

Now, let the system undergo a mass modification $[\Delta M]$ and stiffness modification $[\Delta K]$ consecutively. Assume that we want both systems to have identical dynamic behavior (R1 and R2). The following equations must then be satisfied.

$$\left\{-\omega_k^2\left([M]+[\Delta M]\right)+[K]\right\}\{\psi_k\}=\{0\}, \qquad k=1,\ldots,n \tag{4}$$

$$\left\{-\omega_k^2[M]+([K]+[\Delta K])\right\}\{\psi_k\}=\{0\}, \qquad k=1,\ldots,n \tag{5}$$

Combination of Eqs. (4) and (5) yields the necessary and sufficient set of relations to express identical dynamic behavior for the two types of modifications.

$$\left(-\omega_k^2[\Delta M]-[\Delta K]\right)\{\psi_{k,o}\}=\{0\}, \qquad k=1,\ldots,n \tag{6}$$

havior by means of stiffness (mass) modifications. Because Eq. (8) is setwise uncoupled, another important aspect is the conclusion that a mass (stiffness) modification δm_{ij} (δk_{ij}) will only yield equivalent δk_{ir} (δm_{ir}), $r=1,\ldots,n$, changes. Related to the model-updating process, this means that large unexplainable changes in the stiffness values in a row (or column) may be caused by inaccuracies on the corresponding mass (stiffness) element.

2) There is no equivalent model with a modified stiffness distribution for the original structure ($\delta m_{ij}=0$, $i=1,\ldots,n$, $j=1,\ldots,n$), because this condition yields a system of homogeneous equations with a nonsingular coefficient matrix.

3) In general, an unsymmetric modified stiffness matrix is obtained as equivalent to an arbitrarily mass modified model. This means that no solution exists if the stiffness matrix should remain physically meaningful: $([K]+[\Delta K])^t=([K]+[\Delta K])$. There is also no guarantee that the stiffness elements on the main diagonal will remain positive (positive definite stiffness matrix). It is even possible that stiffness coefficients are generated between points that are not physically connected.

More explicitly, in matrix notation:

$$\begin{bmatrix} [\Delta K] & [0] & [0] & \cdots & [0] \\ [0] & [\Delta K] & [0] & \cdots & [0] \\ [0] & [0] & [\Delta K] & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & [\Delta K] \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_n \end{Bmatrix} = \begin{bmatrix} -\omega_1^2[\Delta M] & [0] & [0] & \cdots & [0] \\ [0] & -\omega_2^2[\Delta M] & [0] & \cdots & [0] \\ [0] & [0] & -\omega_3^2[\Delta M] & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & -\omega_n^2[\Delta M] \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_n \end{Bmatrix} \tag{7}$$

Denoting the elements of $[\Delta M]$ as δm_{ij}, the elements of $[\Delta K]$ as δk_{ij}, and if the following notation is introduced

$$[\psi]=[\psi_1 \quad \psi_2 \quad \psi_3 \quad \ldots \quad \psi_n]$$

and

$$[\Lambda]=\begin{bmatrix} \omega_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \omega_3^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_n^2 \end{bmatrix}$$

it is possible to rearrange Eq. (7) to obtain

$$\begin{bmatrix} [\psi]^t & [0] & [0] & \cdots & [0] \\ [0] & [\psi]^t & [0] & \cdots & [0] \\ [0] & [0] & [\psi]^t & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & [\psi]^t \end{bmatrix} \begin{Bmatrix} \delta k_{11} \\ \vdots \\ \delta k_{1n} \\ \delta k_{21} \\ \vdots \\ \delta k_{2n} \\ \cdots \\ \delta k_{n1} \\ \vdots \\ \delta k_{nn} \end{Bmatrix} = \begin{bmatrix} -[\Lambda][\psi]^t & [0] & [0] & \cdots & [0] \\ [0] & -[\Lambda][\psi]^t & [0] & \cdots & [0] \\ [0] & [0] & -[\Lambda][\psi]^t & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & -[\Lambda][\psi]^t \end{bmatrix} \begin{Bmatrix} \delta m_{11} \\ \vdots \\ \delta m_{1n} \\ \delta m_{21} \\ \vdots \\ \delta m_{2n} \\ \cdots \\ \delta m_{n1} \\ \vdots \\ \delta m_{nn} \end{Bmatrix} \tag{8}$$

Equation (8) constitutes a set of linear relations from which the vector of R1-R2 equivalent stiffness (mass) modifications can be computed, if mass (stiffness) changes are introduced. Several interesting remarks can be made.

1) Because the modal matrix $[\psi]$ is a nonsingular matrix, a unique solution will exist for whatever mass (stiffness) modification is considered, in order to obtain identical dynamic be-

4) For the purpose of model updating, the following important conclusion holds. Assume that the mass modified model is correct and yields identical dynamic behavior to the experimental modal survey test results. Consider a model-update run, starting from the initial (unmodified) finite-element model. According to remark 3, a perfect model solution cannot be obtained when only stiffness variables are considered

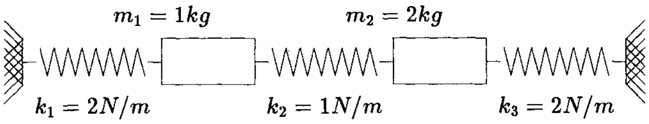

Fig. 1 Mass-spring system 1.

and parameter modifications are required to yield symmetric stiffness changes. On the other hand, if it is only required to match a limited number of modes and eigenfrequencies within a certain frequency band, it is possible to obtain an equivalent model for this reduced set of requirements, by means of stiffness changes alone. However, the dynamic behavior description outside this frequency band will lose accuracy because the number of available parameters is insufficient to satisfy requirements for all of the dynamic characteristics outside this frequency band (remark 3). This has important implications if we want to apply modal synthesis or sensitivity methods to this updated model to evaluate design changes or substructure coupling effects. That is, in general, the modes outside the frequency band of interest have an influence on the accuracy of these predictions.[10] Also, transient-response calculations of these models will yield erroneous results if the frequency content of the input force is broader than the frequency band used for updating. These remarks are even more important if such a model-updating process is performed when not all modes within the frequency band of interest are measured (for example, because of an incomplete test setup). Identical conclusions are obtained for mass modifications when a modified stiffness model is considered. Here we can remark that starting from a lumped mass model (nonconsistent mass matrix), we would end up with an unsymmetric, fully populated mass matrix (nonzero off diagonal elements), and introducing requirements to keep the off-diagonal terms equal to zero would distort the solution even more than in the case of stiffness modificaitons, due to the higher number of additional restrictions.

So, in the context of model updating, it can be concluded:

1) In order to include the perfect model solution for a model-updating technique in the solution space in which updating takes place (space depending on the updating parameters set selected), both stiffness and mass updating must be considered.

2) However, if we limit ourselves to match a certain number of modes and frequencies, an acceptable solution can be obtained by only working on stiffness changes (or only on mass changes). One should be aware, however, that the increase of accuracy within the considered frequency band will involve a corresponding loss of accuracy outside this frequency band of interest. The effect will be more pronounced, if the original mass matrix (or stiffness matrix) is known with less accuracy.

In order to illustrate this discussion, a two-degree-of-freedom spring-mass system is considered (Fig. 1).

The mass and stiffness matrices $[M_1]$ and $[K_1]$ are

$$[M_1] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \qquad [K_1] = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

If a mass modification of $\delta m_{11} = +1kg$ is considered, the modified system 2 will be described by its structural matrices $[M_2]$ and $[K_2]$:

$$[M_2] = [M_1] + [\Delta M] = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad [K_2] = [K_1] = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

System 2 has eigenfrequencies and modeshapes given in Table 1.

In order to find the stiffness modified structure 3, equivalent to structure 2, Eq. (8) is applied:

$$\begin{bmatrix} \psi_{11} & \psi_{21} \\ \psi_{12} & \psi_{22} \end{bmatrix} \begin{Bmatrix} \delta k_{21} \\ \delta k_{22} \end{Bmatrix} = \begin{bmatrix} -\omega_1^2\psi_{11} & -\omega_1^2\psi_{21} \\ -\omega_2^2\psi_{12} & -\omega_2^2\psi_{22} \end{bmatrix} \begin{Bmatrix} \delta m_{21} \\ \delta m_{22} \end{Bmatrix}$$

Introducing numerical values and solving this system of equations yields

$$\begin{cases} \delta k_{12} = 0.5 \\ \delta k_{11} = -1.5 \\ \delta k_{21} = 0.0 \\ \delta k_{22} = 0.0 \end{cases}$$

So, the equivalent stiffness modified structure 3 will have the following characteristics:

$$[M_3] = [M_1] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad [K_3] = [K_1] + [\Delta K] = \begin{bmatrix} 1.5 & -0.5 \\ -1 & 3 \end{bmatrix}$$

This example shows what has been concluded previously. That is, the stiffness matrix may become nonsymmetric and lose physical significance.

The next step is to introduce requirement R3. Although both mass and stiffness modified systems have the same eigenfrequencies and eigenvectors, they may have different modal masses. Indeed, $[\psi]^t[M_2][\psi] = [\psi]^t[M_3][\psi]$ only if

$$[\psi]^t[\Delta M][\psi] = [0] \tag{9}$$

However, Eq. (9) does not hold automatically, as may be illustrated by working out the previous example.

Both modified (assumed identical from a dynamic point of view) models 2 and 3 are scaled to unit modal mass by calculation of proper scaling factors a and b for mode 1 and 2, respectively.

Mode 1 (mass modified structure):

$$a^2 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}^t \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 1 \Rightarrow a = 0.5 \Rightarrow \{\psi_1\} = \begin{Bmatrix} 0.5 \\ 0.5 \end{Bmatrix}$$

Mode 1 (stiffness modified structure):

$$a^2 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}^t \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 1 \Rightarrow a = \sqrt{1/3} \Rightarrow \{\psi_1\} = \begin{Bmatrix} \sqrt{1/3} \\ \sqrt{1/3} \end{Bmatrix}$$

Mode 2 (mass modified structure):

$$b^2 \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}^t \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 1 \Rightarrow b = 0.5 \Rightarrow \{\psi_2\} = \begin{Bmatrix} 0.5 \\ -0.5 \end{Bmatrix}$$

Mode 2 (stiffness modified structure):

$$b^2 \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}^t \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = 1 \Rightarrow b = \sqrt{1/3} \Rightarrow \{\psi_2\} = \begin{Bmatrix} \sqrt{1/3} \\ -\sqrt{1/3} \end{Bmatrix}$$

So in fact both systems would have a different dynamic behavior because according to Eqs. (1) and (2),

$$[H_{\text{mass modified}}(\omega)] = 3/4[H_{\text{stiffness modified}}(\omega)]$$

In general, this relation will be more complex because a and b are not necessarily identical. Indeed, for a general two-dimen-

Table 1 Dynamic characteristics of system 2

Frequency	Mode shapes
$\omega_1 = 1$ rad/s	$\psi_{11} = 1$ $\psi_{21} = 1$
$\omega_2 = \sqrt{2}$ rad/s	$\psi_{12} = -1$ $\psi_{22} = 1$

sional system with an assumed mass matrix 2 (stiffness modified structure),

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

and mass matrix 3 (mass modified structure),

$$\begin{bmatrix} m_1 + \delta m_1 & 0 \\ 0 & m_2 + \delta m_2 \end{bmatrix}$$

Both matrices have identical eigenvectors: ψ_1 and ψ_2, which are scaled to unit modal mass using mass matrix 2 ($a = 1$, $b = 1$ for structure 2). Scaling to unit modal mass using mass matrix 3 yields

Mode 1:

$$a^2 = \frac{1}{\psi_{11}^2(m_1 + \delta m1) + \psi_{21}^2(m_2 + \delta m_2)} \neq 1 \qquad \text{in general}$$

Mode 2:

$$b^2 = \frac{1}{\psi_{12}^2(m_1 + \delta m1) + \psi_{22}^2(m_2 + \delta m_2)} \neq 1 \qquad \text{in general}$$

So the scaled modes would be $a\{\psi_1\}$ and $b\{\psi_2\}$. Note that $a \neq b$ unless $\psi_{11}^2 = \psi_{12}^2$ and $\psi_{21}^2 = \psi_{22}^2$, which occurred in the specific numerical example.

Thus, it is possible to conclude in general that n-dimensional systems cannot have identical dynamic behavior when modeling mass modification changes by means of equivalent stiffness changes, because

$$[H_2(\omega)] = \sum_{k=1}^{n} \left[\frac{q_k\{\psi_k\}\{\psi_k\}^t}{(j\omega - \lambda_k)} + \frac{q_k^*\{\psi_k^*\}\{\psi_k^*\}^t}{(j\omega - \lambda_k^*)} \right] \neq$$

$$[H_3(\omega)] = \sum_{k=1}^{n} \left[\frac{a_k^2 q_k\{\psi_k\}\{\psi_k\}^t}{(j\omega - \lambda_k)} + \frac{(a_k^*)^2 q_k^*\{\psi_k^*\}\{\psi_k^*\}^t}{(j\omega - \lambda_k^*)} \right]$$

Transient response for both structures consists of an excitation of the same modes and the same frequencies, but modal participation for each of the modes will be different for the same input spectrum. In fact, the response spectrum will be distorted (it is not simply a proportional factor that can be corrected for). This yields the conclusion that the use of stiffness changes to correct mass matrix errors during model updating results in distortion of the frequency-response functions. Therefore, when employing model-updating techniques, modal mass changes should be checked carefully. In addition, including the restriction that the total sum of the mass elements remains constant will improve the global response level for a prescribed input, but will not guarantee that all modes are excited correctly (distortion on the participation coefficients).

In conclusion, in order that model-updating procedures are developed consistently, it is advisable to include constraints requiring that the finite-element modal masses converge to the measured modal masses. In fact, it is not difficult to achieve this requirement using updating procedures that account for mass orthogonality conditions which reduce off-diagonal orthogonality terms to an acceptable level. Indeed, after scaling measured modes to unit modal mass, identical analytical modal masses are obtained forcing the main diagonal coefficients of the mass orthogonality to 1.

Unfortunately, in practice, experimentally derived modal masses are considered as the least accurate information from a modal analysis test. The reasons for this are discussed extensively in Ref. 11, together with some techniques to improve the experimental modal mass estimates. Therefore, many model-updating schemes do not take into account this additional information, giving more credit to the analytically obtained modal masses.

Certainly in one way or another, the user of updating procedures must be aware of these problems and should evaluate updating results critically. Otherwise, incorrect conclusions may result when applying modal synthesis (For example, for evaluating design alternatives), transient-response calculations, sensitivity analysis, . . . on the updated finite-element model.

IV. Equivalence of Finite-Element Models

The problem of identification of equivalent finite-element models is related to the discussion of Sec. III. However, the question is more explicitly stated now as "Is it possible to find equivalent models for any initial finite-element model from the point of view of identical dynamic behavior?" The answer to this question yields conclusions about the uniqueness of structural dynamic behavior characteristics.

Assume a finite-element model 1: $[K]$, $[M]$, having dynamic characteristics: ω_k, ψ_k ($k = 1, \ldots, n$):

$$(-\omega_k^2[M] + [K])\{\psi_k\} = \{0\}, \qquad k = 1, \ldots, n \tag{10}$$

Other finite-element models predicting the same dynamic behavior (requirements R1 and R2) can be identified from

$$\left\{-\omega_k^2([M] + [\Delta M]) + ([K] + [\Delta K])\right\}\{\psi_k\} = \{0\} \qquad k = 1, \ldots, n \tag{11}$$

Combination of Eqs. (10) and (11) yields the necessary and sufficient conditions for identical dynamic behavior. Using the same procedure as in Sec. III yields

$$\begin{bmatrix} [\psi]^t & [0] & [0] & \cdots & [0] \\ [0] & [\psi]^t & [0] & \cdots & [0] \\ [0] & [0] & [\psi]^t & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & [\psi]^t \end{bmatrix} \begin{Bmatrix} \delta k_{11} \\ \vdots \\ \delta k_{1n} \\ \delta k_{21} \\ \vdots \\ \delta k_{2n} \\ \cdots \\ \delta k_{n1} \\ \vdots \\ \delta k_{nn} \end{Bmatrix} = \begin{bmatrix} [\Lambda][\psi]^t & [0] & [0] & \cdots & [0] \\ [0] & [\Lambda][\psi]^t & [0] & \cdots & [0] \\ [0] & [0] & [\Lambda][\psi]^t & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & [0] & \cdots & [\Lambda][\psi]^t \end{bmatrix} \begin{Bmatrix} \delta m_{11} \\ \vdots \\ \delta m_{1n} \\ \delta m_{21} \\ \vdots \\ \delta m_{2n} \\ \cdots \\ \delta m_{n1} \\ \vdots \\ \delta m_{nn} \end{Bmatrix} \tag{12}$$

All elements δk_{ij} and δm_{ij} are considered to be independent variables. Therefore, the dimension of the solution space is $(2n^2 - n^2) = n^2$. The next step is to consider the physically acceptable case where symmetry of the mass and stiffness matrices is required. That is, $\delta k_{ij} = \delta k_{ji}$ and $\delta m_{ij} = \delta m_{ji}$. Introducing these constraints into Eq. (12) gives the result

$$
\begin{bmatrix}
\psi_{11} & \psi_{21} & \cdots & \cdots & \psi_{n1} & 0 & \cdots & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\psi_{1n} & \psi_{2n} & \cdots & \cdots & \psi_{nn} & 0 & \cdots & \cdots & 0 & \cdots & 0 \\
0 & \psi_{11} & 0 & \cdots & 0 & \psi_{21} & \psi_{31} & \cdots & \psi_{n1} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \psi_{1n} & 0 & \cdots & 0 & \psi_{2n} & \psi_{3n} & \cdots & \psi_{nn} & \cdots & 0 \\
0 & 0 & \psi_{11} & \cdots & 0 & 0 & \psi_{21} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \psi_{1n} & \cdots & 0 & 0 & \psi_{2n} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \psi_{11} & 0 & 0 & \cdots & \psi_{21} & \cdots & \psi_{n1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \psi_{1n} & 0 & 0 & \cdots & \psi_{2n} & \cdots & \psi_{nn}
\end{bmatrix}
\begin{Bmatrix}
\delta k_{11} \\ \vdots \\ \delta k_{1n} \\ \delta k_{22} \\ \vdots \\ \delta k_{2n} \\ \vdots \\ \delta k_{nn}
\end{Bmatrix}
$$

$$
= \begin{bmatrix}
\omega_1^2\psi_{11} & \omega_1^2\psi_{21} & \cdots & \cdots & \omega_1^2\psi_{n1} & 0 & \cdots & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\omega_n^2\psi_{1n} & \omega_n^2\psi_{2n} & \cdots & \cdots & \omega_n^2\psi_{nn} & 0 & \cdots & \cdots & 0 & \cdots & 0 \\
0 & \psi_{11} & 0 & \cdots & 0 & \omega_1^2\psi_{21} & \omega_1^2\psi_{31} & \cdots & \omega_1^2\psi_{n1} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \omega_n^2\psi_{1n} & 0 & \cdots & 0 & \omega_n^2\psi_{2n} & \omega_n^2\psi_{3n} & \cdots & \omega_n^2\psi_{nn} & \cdots & 0 \\
0 & 0 & \omega_1^2\psi_{11} & \cdots & 0 & 0 & \omega_1^2\psi_{21} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \omega_n^2\psi_{1n} & \cdots & 0 & 0 & \omega_n^2\psi_{2n} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \omega_1^2\psi_{11} & 0 & 0 & \cdots & \omega_1^2\psi_{21} & \cdots & \omega_1^2\psi_{n1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \omega_n^2\psi_{1n} & 0 & 0 & \cdots & \omega_n^2\psi_{2n} & \cdots & \omega_n^2\psi_{nn}
\end{bmatrix}
\begin{Bmatrix}
\delta m_{11} \\ \vdots \\ \delta m_{1n} \\ \delta m_{22} \\ \vdots \\ \delta m_{2n} \\ \vdots \\ \delta m_{nn}
\end{Bmatrix}
\qquad (13)
$$

As inspection of the form of Eq. (13) produces the dimension of the reduced solution space as $[n(n+1)-n^2]=n$. Thus, if these n remaining degrees of freedom are used to specify n supplementary restrictions, a $(n^2+n)\times(n^2+n)$ homogeneous set of equations is obtained. In general, if the n additional equations are assumed to be linearly independent from Eqs. (13), the solution to the problem is unique: $\delta k_{ij}=0$ and $\delta m_{ij}=0$. One logical choice of additional requirements can be the requirements R3. For the general case where a nonsingular coefficient matrix is obtained for the $(n^2+n)\times(n^2+n)$ homogeneous set of equations, it may be concluded that the corresponding finite-element model is uniquely defined. The complementary set of equations for the modal mass criterion can be defined as

$$\{\psi_k\}^t([M]+[\Delta M])\{\psi_k\}^t = \{\psi_k\}^t[M]\{\psi_k\}, \qquad k=1,\ldots,n$$

or

$$\{\psi_k\}^t[\Delta M]\{\psi_k\} = \{0\}, \qquad k=1,\ldots,n$$

More explicitly,

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(\psi_{ik}\psi_{jk}\delta m_{ij})=0, \qquad k=1,\ldots,n$$

Taking into account the symmetry condition $\delta m_{ij}=\delta m_{ji}$ yields

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(2\psi_{ik}\psi_{jk}\delta m_{ij})+\sum_{i=1}^{n}\psi_{ii}^2\delta m_{ii}=0, \qquad k=1,\ldots,n \quad (14)$$

For the case of a lumped mass system, the dimension of the solution space of Eq. (13) is further reduced to a dimension of $[n(n+1)/2+n-n^2]=(3n-n^2)/2$. This means that, in general, a solution for Eq. (13) would only be found for a two-degree-of-freedom system. What can also be concluded from the last equation is that for a lumped mass representation, an equivalent matrix formulation cannot be derived without changing the modal masses, and thus the modal participation for each mode. In the chain of reasoning of Sec. III, it may be seen that distortion of the calculated frequency-response curves also occurs.

For the sake of clarity, the numerical example of Sec. III is considered again. The set of relations for the lumped-mass, two-degree-of-freedom system, in accordance with Eq. (12) becomes

$$\begin{bmatrix}\psi_{11} & \psi_{21}\\ \psi_{12} & \psi_{22}\end{bmatrix}\begin{Bmatrix}\delta k_{11}\\ \delta k_{12}\end{Bmatrix} = \begin{bmatrix}\omega_1^2\psi_{11} & \omega_1^2\psi_{21}\\ \omega_2^2\psi_{12} & \omega_2^2\psi_{22}\end{bmatrix}\begin{Bmatrix}\delta m_{11}\\ 0\end{Bmatrix}$$

$$\begin{bmatrix}\psi_{11} & \psi_{21}\\ \psi_{12} & \psi_{22}\end{bmatrix}\begin{Bmatrix}\delta k_{21}\\ \delta k_{22}\end{Bmatrix} = \begin{bmatrix}\omega_1^2\psi_{11} & \omega_1^2\psi_{21}\\ \omega_2^2\psi_{12} & \omega_2^2\psi_{22}\end{bmatrix}\begin{Bmatrix}0\\ \delta m_{22}\end{Bmatrix}$$

The dimension of the solution space (same eigenfrequencies, same modeshapes) is reduced to 1, taking into account the symmetry requirements ($\delta k_{12}=\delta k_{21}$).

Indeed, δm_{11} can be chosen freely, for example, and δk_{11} and δk_{12} are solved from the first two equations. The two last equations allow the determination of δk_{22} and δm_{22}.

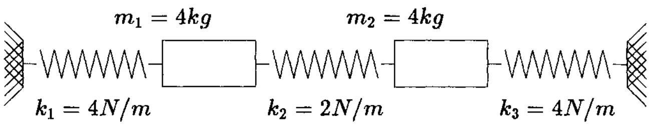

Fig. 2 Equivalent mass-spring system.

Introducing numerical values and choosing δm_{11} to be $+2$,

$$\begin{cases} \delta k_{12} = 1.0 \\ \delta k_{11} = 3.0 \\ \delta k_{22} = 3.0 \\ \delta m_{22} = 2.0 \end{cases}$$

The equivalent system is shown in Fig. 2.

As expected, the system has the same eigenfrequencies and mode shapes but the modal masses changed in a proportional way (for the same scaling of the mode shapes).

In reality, there are many zero stiffnesses and masses that should remain unchanged for obvious physical reasons.[5] It is also logical that mass changes for different directions but in the same point are not independent. This means that in reality there is a smaller number of independent variables available for updating.

Considering the purpose of the practical model updating, the main aim is to achieve a good modeling correspondence for a limited number of analytical and experimental characteristics. Thus, there is a reduced number of requirements. The reduction of the number of requirements makes it easier to derive models that match these specifications exactly. However, the following considerations are important:

1) As observed above, when trying to match dynamic behavior within the so-called frequency band of interest, updating the finite-element model in a physically irrelevant way, involves a loss of accuracy for the dynamic behavior outside the considered frequency band. This is important when using the updated model for different types of calculations.

2) Problems can occur in transient-response calculations involving modification analysis techniques that evaluate important structural changes (for example, element stresses are very sensitive to local distortions). Often, it will be very difficult to check the validity of these predictions, especially when using updating procedures as a black box, where no concern is given to the interpretation of the resulting finite-element model.

V. Conclusions

In this study, attention is drawn to the uniqueness aspects of structural dynamic behavior. Based on the present theoretical study, it can be concluded that

1) Mass *and* stiffness properties should be updated simultaneously to obtain qualitatively good update results.

2) In the majority of update runs, an increase of accuracy within the frequency band of interest may be accompanied by a corresponding loss of accuracy outside the considered frequency band.

3) Modal mass changes should be checked and evaluated carefully during model updating.

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